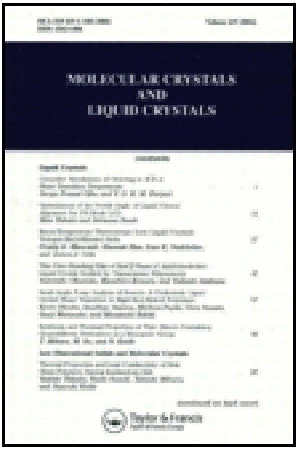

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The Reflectivity of a Liquid Crystal Cell in a Surface Plasmon Experiment

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The Reflectivity of a Liquid Crystal Cell in a Surface Plasmon Experiment†

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This paper provides the mathematical background on which the previous paper entitled: Determination of the Surface Tilt Angle by ATR, is based. The algebra was considered too lengthy to be incorporated as an appendix to that paper.

I INTRODUCTION

This paper accompanies the previous paper entitled: Determination of the Surface Tilt Angle by Attenuated Total Reflection.[1] It contains the derivation of the reflectivity matrix on which the calculations were based. The algebra was considered too lengthy to be incorporated as an appendix to the previous paper.

The geometry is as follows: A *p*-polarized beam generates an evanescent wave in a thin gold film covered with a thin SiO_2 aligning layer. The aligning layer controls the alignment of the liquid crystal material such that its director—the optic axis—is in the plane of polarization. Without a bias field the liquid crystal layer is homogeneous and the director is parallel to the surface. As a field is applied the director tilts but remains in the plane of polarization. Therefore, only the extraordinary ray needs to be considered which greatly reduces the complexity of the problem.

In the next section we derive the dispersion relations needed in the development of the matrix equation in Section III.

II DISPERSION RELATIONS

The coordinate system is defined in Figure 1. Consider *p*-polarized waves of the form:

† Presented at the Eighth International Liquid Crystal Conference, Kyoto, July 1980.

$$E_x^+ = E_x^{0+} \exp i(\omega t - (k_x \cdot x + k_z^+ \cdot z))$$
$$E_z^+ = E_z^{0+} \exp i(\omega t - (k_x \cdot x + k_z^+ \cdot z))$$
$$E_y^+ = 0. \tag{1}$$

To distinguish between incident and reflected waves we shall use the superscripts + and −, respectively, wherever needed. See Figure 2. The starting point is the pair of Maxwell's equations:

$$\nabla \times E = -\frac{1}{c}\dot{H}$$
$$\nabla \times H = \frac{1}{c}\dot{D} \tag{2}$$

for $\mu = 1$.

From (2) and (1)

$$\frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} = -\frac{1}{c}\dot{H}_y$$

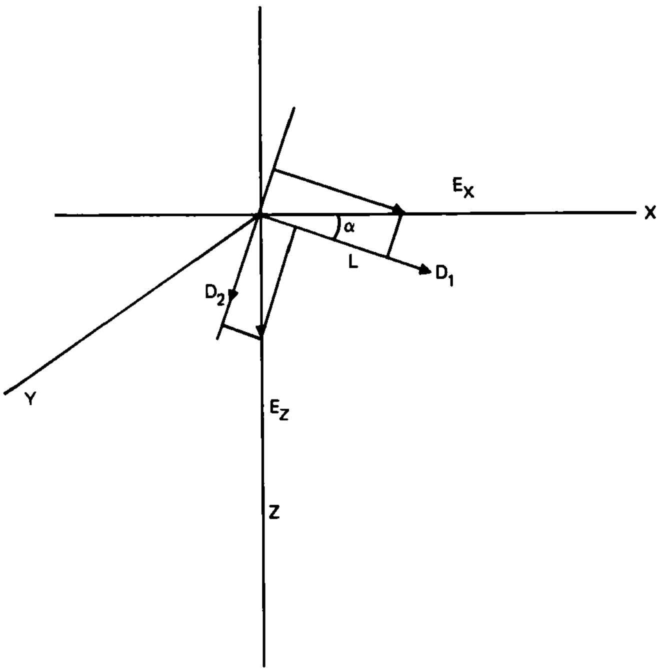

FIGURE 1 Coordinate system. xy is the surface. xz is plane of polarization. The director L coincides with the optical axis D_1. α is the tilt angle.

$$-\frac{\partial H_y}{\partial z} = \frac{1}{c}\dot{D}_x$$

$$\frac{\partial H_y}{\partial x} = \frac{1}{c}\dot{D}_z. \tag{3}$$

From geometry in Figure 1

$$D_1 = \epsilon_1(E_x \cos\alpha + E_z \sin\alpha)$$

$$D_2 = \epsilon_2(-E_x \sin\alpha + E_z \cos\alpha)$$

$$D_x = D_1 \cos\alpha - D_2 \sin\alpha$$

$$D_z = D_1 \sin\alpha + D_2 \cos\alpha$$

$$D_x = E_x(\epsilon_1 \cos^2\alpha + \epsilon_2 \sin^2\alpha) + E_z(\epsilon_1 - \epsilon_2)\sin\alpha\cos\alpha$$

$$D_z = E_z(\epsilon_1 \sin^2\alpha + \epsilon_2 \cos^2\alpha) + E_x(\epsilon_1 - \epsilon_2)\sin\alpha\cos\alpha. \tag{4}$$

Abbreviate (4) to:

$$D_x = A\cdot E_x + B\cdot E_z$$

$$D_z = C\cdot E_z + B\cdot E_x. \tag{4.1}$$

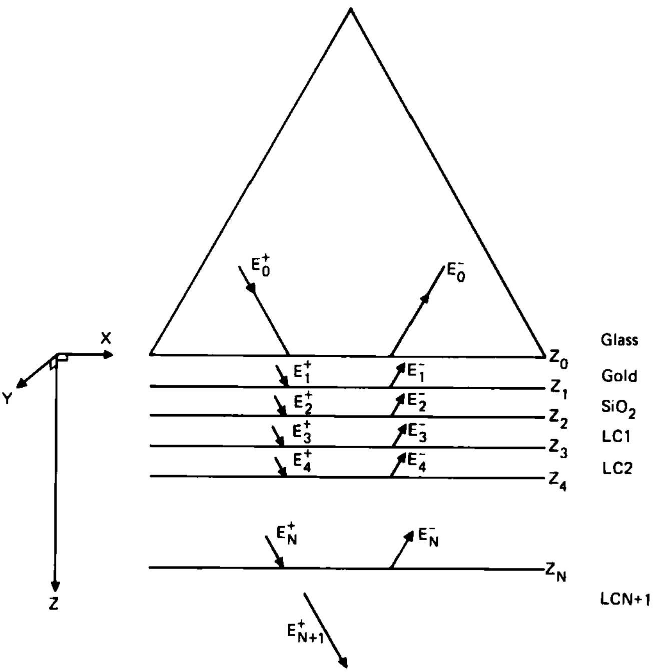

FIGURE 2 Schematic diagram for an ATR experiment on an operational LC cell.

Substitute (4.1) in (3)

$$\left(\frac{\omega}{c}\right) H_y = k_z^+ \cdot E_x - k_x \cdot E_z$$

$$-k_z \cdot H_y = \left(\frac{\omega}{c}\right)(C \cdot E_z + B \cdot E_x)$$

$$k_z^+ \cdot H_y = \left(\frac{\omega}{c}\right)(A \cdot E_x + B \cdot E_z). \tag{5}$$

Elimination of H_y, E_x and E_z leads to

$$k_x^2 \cdot A + k_z^{+2} \cdot C + 2k_x k_z^+ B = \left(\frac{\omega}{c}\right)^2 (AC - B^2) = \left(\frac{\omega}{c}\right)^2 \cdot \epsilon_1 \cdot \epsilon_2. \tag{6}$$

Solve for k_z^+

$$k_z^+ = -k_x \cdot \frac{B}{C} \pm \frac{\sqrt{\epsilon_1 \epsilon_2}}{C} \cdot \sqrt{\left(\frac{\omega}{c}\right)^2 \cdot C - k_x^2}. \tag{7}$$

Equation (7) is the dispersion relation used for numerical work. However, (6) can be written in the alternate form:

$$\frac{(k_x \cos\alpha + k_z \sin\alpha)^2}{\epsilon_2} + \frac{(k_x \sin\alpha - k_z \cos\alpha)^2}{\epsilon_1} = \left(\frac{\omega}{c}\right)^2$$

write:

$$k_x \cos\alpha + k_z \sin\alpha = k_1$$

$$k_z \cos\alpha - k_x \sin\alpha = k_2$$

$$\frac{k_1^2}{\epsilon_2} + \frac{k_2^2}{\epsilon_1} = \left(\frac{\omega}{c}\right)^2 \tag{8}$$

(8) has the form familiar from the isotropic case.

For the reflected wave the wavefunctions are of the type:

$$E_x^- = E_x^{0-} \exp i(\omega t - (k_x x - k_z^- \cdot z))$$

etc. Following the same procedure, one arrives at:

$$k_z^- = k_x \cdot \frac{B}{C} \pm \frac{\sqrt{\epsilon_1 \epsilon_2}}{C} \sqrt{\left(\frac{\omega}{c}\right)^2 \cdot C - k_x^2}. \tag{9}$$

In general the square root will be imaginary. To make the amplitude decay in the z-direction the $-$ sign is appropriate in (7), the $+$ sign in (9). The relation (8), written in the principal axis is valid for both incident and reflected wave.

In the following, relations between H_y^+ and E_x^+ and H_y^- and E_x^- are needed. From (5) and (7) one derives for the incident wave:

$$H_y^+ = -\left(\frac{\omega}{c}\right)\cdot\frac{\sqrt{\epsilon_1\epsilon_2}}{\sqrt{\left(\frac{\omega}{c}\right)^2\cdot C - k_x^2}}\cdot E_x^+ \tag{10}$$

and for the reflected wave:

$$H_y^- = \left(\frac{\omega}{c}\right)\cdot\frac{\sqrt{\epsilon_1\epsilon_2}}{\sqrt{\left(\frac{\omega}{c}\right)^2\cdot C - k_x^2}}\cdot E_x^- \tag{11}$$

abbreviate

$$u = \frac{\sqrt{\epsilon_1\epsilon_2}}{\sqrt{\left(\frac{\omega}{c}\right)^2\cdot C - k_x^2}} \tag{12}$$

$$H_y^+ = -\left(\frac{\omega}{c}\right)\cdot u\cdot E_x^+$$

$$H_y^- = \left(\frac{\omega}{c}\right)\cdot u\cdot E_x^-. \tag{12a}$$

Note that for $\alpha = 0$, $B = 0$ and $C = \epsilon_2$ and we recover the familiar expressions

$$H_y^+ = \frac{\omega}{c}\cdot\frac{\epsilon_1}{k_z^+}E_x^+$$

$$H_y^- = \frac{\omega}{c}\cdot\frac{\epsilon_1}{k_z^-}E_x^-. \tag{13}$$

III REFLECTIVITY OF A MULTILAYERED BIREFRINGENT MEDIUM

Figure 2 defines the parameters used. Similar schemes are used by Wolter[2] and Heavens.[3] From boundary conditions at $z = z_0$:

$$E_{0x}^+ \exp - ik_{0z}\cdot z_0 + E_{0x}^- \exp + ik_{0z}\cdot z_0 = E_{1x}^+ \exp - ik_{1z}\cdot z_0 + E_{1x}^- \exp + ik_{1z}\cdot z_0$$

$$H_{0y}^+ \exp - ik_{0z}\cdot z_0 + H_{0y}^- \exp + ik_{0z}\cdot z_0 = H_{1y}^+ \exp - ik_{1z}\cdot z_0 + H_{1y}^- \exp + ik_{iz}\cdot z_0. \tag{14}$$

Eliminate H_y^+ and H_y^- using Eq. (12) and solving for E_{0x}^+ and E_{0x}^- one finds:

$$\begin{pmatrix} E_{0x}^+ \exp - ik_{0z}\cdot z_0 \\ \\ E_{0x}^- \exp + ik_{0z}\cdot z_0 \end{pmatrix} = \begin{vmatrix} \frac{u_0+u_1}{2u_0} & \frac{u_0-u_1}{2u_0} \\ \\ \frac{u_0-u_1}{2u_0} & \frac{u_0+u_1}{2u_1} \end{vmatrix} \times \begin{pmatrix} E_{1x}^+ \exp - ik_{1z}\cdot z_0 \\ \\ E_{1x}^- \exp + ik_{1z}\cdot z_0 \end{pmatrix} \quad (15)$$

Similarly at $z = z_1$:

$$\begin{pmatrix} E_{1x}^+ \exp - ik_{1z}\cdot z_1 \\ \\ E_{1x}^- \exp + ik_{1z}\cdot z_1 \end{pmatrix} = \begin{vmatrix} \frac{u_1+u_2}{2u_1} & \frac{u_1-u_2}{2u_0} \\ \\ \frac{u_1-u_2}{2u_1} & \frac{u_1+u_2}{2u_1} \end{vmatrix} \times \begin{pmatrix} E_{2x}^+ \exp - ik_{2z}\cdot z_1 \\ \\ E_{2x}^- \exp + ik_{2z}\cdot z_1 \end{pmatrix} \quad (33)$$

which is easily transformed to:

$$\begin{pmatrix} E_{1x}^+ \exp - ik_{1z}\cdot z_0 \\ \\ E_{1x}^- \exp + ik_{1z}\cdot z_0 \end{pmatrix} = \begin{vmatrix} \frac{u_1+u_2}{2u_1} \exp + ik_{1z}d_1 & \frac{u_1-u_2}{2u_1} \exp + ik_{1z}d_1 \\ \\ \frac{u_1-u_2}{2u_1} \exp - ik_{1z}d_1 & \frac{u_1+u_2}{2u_1} \exp - ik_{1z}d_1 \end{vmatrix}$$

$$\times \begin{pmatrix} E_{2x}^+ \exp - ik_{2z}\cdot z_1 \\ E_{2x}^- \exp + ik_{2z}\cdot z_1 \end{pmatrix} \quad (17)$$

Combining (15) and (17)

$$\begin{pmatrix} E_{0x}^+ \exp - ik_{0z}\cdot z_0 \\ E_{0x}^- \exp + ik_{0z}\cdot z_0 \end{pmatrix} = |M_0| \times |M_1| \times \begin{pmatrix} E_{2x}^+ \exp - ik_{2z}\cdot z_1 \\ E_{2x}^- \exp + ik_{2z}\cdot z_i \end{pmatrix} \quad (18)$$

where M_0 and M_1 are the 2×2 matrices in (15) resp. (17) and $d_1 = z_1 - z_0$.

Repeating the process for every interface $z_2, z_3, \ldots, z_N$ one arrives at:

$$\begin{pmatrix} E_{0x}^+ \\ E_{0x}^- \end{pmatrix} = \prod_{n=0}^{n=N} |M_n| \times \begin{pmatrix} E_{N+1}^+ \exp - ik_{N+1,z}\cdot z_N \\ 0 \end{pmatrix} \quad (19)$$

where:

$$|M_n| = \begin{vmatrix} \dfrac{u_n + u_{n+1}}{2u_n} \exp + ik_{n,z} \cdot d_n & \dfrac{u_n - u_{n+1}}{2u_n} \exp + ik_{n,z} \cdot d_n \\ \dfrac{u_n - u_{n+1}}{2u_n} \exp - ik_{n,z} \cdot d_n & \dfrac{u_n + u_{n+1}}{2u_n} \exp - ik_{n,2} \cdot d_n \end{vmatrix}$$

and

$$d_n = z_{n+1} - z_n \text{ and } d_0 = 0$$

arbitrarily $z_0 = 0$.

It is assumed that $E^-_{N+1} = 0$, no reflected wave into the stack. Medium 0 (glass prism), medium 1 (gold film) and medium 2 (aligning layer) are optically isotropic and the functions u are

$$u_i = \frac{\epsilon_i}{k_{iz}} \quad i = 0,1,2.$$

For all other layers the u_i's are given by Eq. (12). If all ϵ and d's are known each matrix is determined and the matrix product can be evaluated. The end result will again be a 2×2 matrix. If this is written as:

$$\begin{vmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{vmatrix}$$

then it is seen that

$$R \equiv \frac{E_0^-}{E_0^+} = \frac{E_{0x}^-}{E_{0x}^+} = \frac{M_{21}}{M_{11}}.$$

Eq. (19) is the basis for the calculation of the ATR reflectivity curves in the previous paper.[1] Evaluation of all ϵ_i and d_i in succession is discussed there.

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3. O. S. Heavens, *Optical Properties of Thin Solid Films*, Dover 1965.